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# Adaptive Radar Signal Processing-The Problem of Exponential Computational Cost

Muralidhar Rangaswamy,

Air Force Research Laboratory/SNHE Hanscom Air Force Base, MA, USA

Email: Muralidhar.Rangaswamy@hanscom.af.mil

**Abstract** — *This paper provides a survey of space-time adaptive processing for radar target detection. Specifically, early work on adaptive array processing from the point of view of maximum signal-to-noise-ratio and minimum mean squared error perspectives are briefly reviewed for motivation. The sample matrix inversion method of Reed, Mallet and Brennan is discussed with attention devoted to its convergence properties. Variants of this approach such as the Kelly GLRT, adaptive matched filter and ACE tests are considered. Extensions to handle the case of non-Gaussian clutter statistics are presented. Current challenges of limited training data support, computational cost, and severely heterogeneous clutter backgrounds are outlined. Implementation and performance issues pertaining to reduced rank and model-based parametric approaches are presented.*

## 1. INTRODUCTION

Signal detection using an array of sensors has offered significant benefits in a variety of applications such as radar, sonar, satellite communications, and seismic systems. Employing an array of sensors overcomes the directivity and beamwidth limitations of a single sensor. Additional gain afforded by an array of sensors leads to improvement in the Signal-to-Noise-ratio, resulting in an ability to place deep nulls in the direction of interfering signals. Finally, a system using an array of sensors affords enhanced reliability compared to a single sensor system. For example, sensor failure in a single sensor system leads to severe degradation in performance whereas sensor failure in an array results in graceful performance degradation.

A problem of considerable importance in this context is the adaptive radar detection of desired targets against a background of interference consisting of clutter, one or more jammers and background noise. The radar receiver front end consists of an array of antenna elements. The received signal is an electromagnetic plane wave impinging on the array manifold. The electromagnetic plane wave induces a voltage at each element of the array, which constitutes the measured data.

Several snapshots of measured data are available in practice. Using the snapshots of data, the problem at hand is to detect desired targets in the presence of interfering signals. An important requirement is that of a constant probability of false alarm. In practice, the interference statistics, the interference spectral characteristics, and the target complex amplitude are unknown. Thus, the problem of adaptive radar target detection in interference is equivalent to the problem of statistical hypothesis testing in the presence of nuisance parameters. Present day computing power permits the use of well-known tools from statistical detection and estimation theory in the radar problem. The Doppler-Wavenumber or angle Doppler spectrum provides a unique representation of a signal in a three dimensional plane. Hence, the problem of space-time adaptive processing (STAP) may also be viewed as a spectrum estimation problem where the two-dimensional Fourier transform of spatio-temporal data affords separation of the desired target from interference. This scenario is described in Figure 1.

## 2. STAP OUTLINE

Typically, a radar transmits a burst of  $N$  pulses in a coherent processing interval. The data measured at the array thus consists of a  $JN \times 1$  complex valued vector, where  $J$  is the number of elements in the array. This corresponds to  $N$  snapshots obtained from the  $J$  element array. Furthermore, since most radars employ a high pulse repetition frequency (PRF), there is a temporal correlation between successive pulses at a given element of the array. Furthermore, the array geometry introduces an element-to-element spatial correlation as shown in Figure 2. Thus in the context of STAP, the unknown interference spectral characteristics correspond to the unknown spatio-temporal correlation or covariance matrix of the  $JN \times 1$  complex-vector under the condition that the data consists of interference alone. Additionally, interference statistics can be either Gaussian or non-Gaussian. In the latter case, all STAP methods would be based on a suitable model for the interference statistics.

The presence of unknown parameters in the problem precludes the use of a uniformly most powerful test for the adaptive target detection problem. This is due to the fact that joint maximization of a likelihood ratio over the domain of unknown parameters is extremely difficult. Hence, ad hoc approaches have been proposed to overcome this problem. Most of the work in the area of STAP is based on the

Gaussian model for the interference. STAP for non-Gaussian interference has received increased attention in recent times.

Succinctly stated, most classical STAP algorithms consist of the following steps depicted in Figure 3.

- (i) Estimate nuisance parameters (interference covariance matrix and target complex amplitude)
- (ii) Form a weight vector based on the inverse covariance matrix
- (iii) Calculate the inner product of the weight vector and the data vector from a cell under test
- (iv) Compare the squared magnitude of the inner product in step (iii) with a threshold determined according to a specified false alarm probability.

Several interesting theoretical interpretations have been offered for the STAP algorithms in the literature. However, from a practical standpoint the key issues include:

- (I) Sufficient target-free training data support to form an estimated interference covariance matrix.
- (II) Non-singular estimated covariance matrix to form the weight vector.
- (III) Computational complexity in forming the weight vector.
- (IV) The ability to maintain a constant false alarm rate (CFAR) and robust detection performance.

### 3. IMPLEMENTATION ISSUES

Early work in the 1960s by Widrow [1] (least squares method), Applebaum [2] (maximum signal-to-noise-ratio criterion) and Howells [3] (sidelobe canceller) suggested the use of feedback loops with an appropriate error criterion to control the convergence of iterative methods for calculating the weight vector in adaptive arrays. However, these methods were slow to converge to the steady-state solution. Fundamental work by Reed, Mallet and Brennan [4] (RMB beamformer) in 1974 showed that the sample matrix inverse method offered considerably better convergence properties compared to the work of Widrow et. al. Key requirements of the RMB beamformer are the availability of at least  $JN$  training data vectors for forming the sample covariance matrix and the availability of  $2JN$  training data vectors to achieve performance within 3 dB of the optimal SNR. Computational complexity of the RMB method is  $O(M^3)$  where  $M=JN$ . A drawback of the RMB approach is the lack of CFAR. Modifications and extensions of this approach to obtain CFAR was the focus of a number of efforts in the 1980s and early 1990s. These resulted in a number of algorithms such as the Kelly-GLRT[5], the adaptive matched filter [6,7], and the adaptive coherence estimator [8-13]. However, training data requirements and computational complexity of the algorithms remain unchanged from that of the RMB beamformer. Performance of all sample covariance based STAP methods degrade in heterogeneous [14-17] and non-Gaussian interference scenarios [18-20]. In

the latter case, this is due to the fact that the sample covariance matrix suffers from significant estimation errors [21-23]. Consequently, a much larger training data support (compared to the Gaussian case) is needed.

On the other hand, collecting sufficient training data depends on system considerations such as bandwidth, frequency agility, and range extent as well as environmental conditions such as the non-homogeneity and non-stationarity of the scanned areas. These factors preclude the collection of large amounts of training data. The problem can become severe with increasing dimensionality. For example, 10 snapshots of data collected from a 32 element antenna array gives rise to the problem of estimating a  $320 \times 320$  covariance matrix. Using the rule of the RMB beamformer, this necessitates the use of 640 target-free training data vectors to estimate the covariance matrix. Assuming an instantaneous RF bandwidth of 200 KHz, the representative training data assumption calls for wide sense stationarity to prevail over a range of 960 Km. Wide sense stationarity of the clutter seldom prevails over such a large region.

Therefore, there is a need to investigate methods, which offer the potential for reducing the computational complexity and the training data requirements for STAP in Gaussian and non-Gaussian interference scenarios. The work of Rangaswamy and Michels [18-20,24,25] provides a useful model-based parametric STAP method, which offers the potential for considerable reduction in training data support and computational complexity. In this method, the data processes are whitened through the use of multi-channel prediction error filters whose coefficients are chosen so as to match the inverse spectral characteristics of the interference. An important feature of this method is the lack of a need to form and invert the interference covariance matrix. Consequently, the limitation of  $O(M^3)$  does not apply here. Furthermore, the use of a low model order filter enables significant reduction in training data support. The low model order approximation has been found to work well in a variety of simulated and real data scenarios. Figure 4 provides a brief overview of the model based parametric method using prediction error filters. The model based parametric method provides excellent performance in both Gaussian [25-27] and non-Gaussian interference scenarios [18-20 and references therein]. Other methods such as the cross spectral metric (CSM) [28], auxiliary vector method (AVM) [29], reduced dimension STAP [30], and multistage Wiener filter (MWF) [31] have been proposed for reducing the computational complexity and training data support requirements. A block diagram of these methods is shown in Figure 4. Additional reduced dimension STAP methods include element-space, beam-space pre-Doppler and post-Doppler techniques[32] and the principal components inverse (PCI) [33] and eigencanceller [34, 35] approaches. An important requirement of these methods is that the reduced-dimension weight vector span the clutter subspace and the signal

subspace. A block diagram of reduced-rank STAP methods is shown in Figure 5.

Many of these methods are able to reduce only the computational complexity requirement since they still require that the estimated covariance matrix have full rank. Furthermore, the performance of the low rank methods severely degrades in non-Gaussian interference scenarios. Another point of note is that most reduced rank STAP methods fail to maintain CFAR in both Gaussian and non-Gaussian interference scenarios. CFAR of reduced dimension methods is a subject of ongoing investigation.

#### 4. CURRENT CHALLENGES AND OPEN PROBLEMS

Advances in system hardware permit the development of large arrays processing a large number of pulses in a CPI. Furthermore, operational scenarios get increasingly complex due to their highly composite nature leading to severe spatio-temporal clutter non-stationarity. Systems considerations such as bandwidth, frequency agility, internal clutter motion, aircraft crabbing, conformal arrays, spaceborne platforms, and bistatic geometry further exacerbate the clutter nonstationarity. Signal contamination of STAP training data leads to target cancellation. These effects call for efficient STAP methods to handle the following:

- (i) Operation in non-stationary, heterogeneous clutter backgrounds (see [36] for details).
- (ii) Reduced training data support for estimation of interference statistics and spectral characteristics (see [25-31,33] for possible approaches).
- (iii) Performance analysis including operational effects-platform velocity, aircraft crab angle, channel mismatch, mutual coupling between the elements of the antenna array.
- (iv) Computational cost reduction.
- (v) CFAR in Gaussian and non-Gaussian interference scenarios using reduced dimension STAP.
- (v) Robust STAP receiver design.
- (vi) Dense target environments (see [15, 36, 37] for details).

Operational scenarios are fiercely composite. Additionally, unknown parameters of pertaining to the clutter statistics need to be estimated online in an adaptive fashion. The corresponding STAP processor for each scenario must then be implemented. Techniques for sensing changes in the clutter statistics need to be employed and the STAP processor must be suitably adapted. Finally, automating the process of sensing and STAP needs to be carried out. Thus, the problem leads to exponential computational cost with increasing system dimension.

#### REFERENCES

- [1] B. Widrow, P.E. Mantey, L.J. Griffiths, and B.B. Goode, "Adaptive antenna systems," *Proceedings of the IEEE*, Vol. 55, December 1967
- [2] S.P. Applebaum, "Adaptive Arrays," Syracuse University Research Corporation, Rep. SU-SEL-66-12, Tech Report 6764-6, December 1966
- [3] P.W. Howells, "Intermediate frequency sidelobe canceller," U.S. Patent 3202990, August 24, 1965
- [4] I.S. Reed, J.D. Mallett, and L.E. Brennan, "Rapid convergence rate in adaptive arrays," *IEEE Trans. on Aerospace and Electronic Systems*, Vol. 10, September 1974, pp. 853-863
- [5] E.J. Kelly, "An adaptive detection algorithm," *IEEE Trans. on Aerospace and Electronic Systems*, Vol. 22, 1986, pp. 115-127
- [6] F.C. Robey, D.R. Fuhrmann, E.J. Kelly, and R. Nitzberg, "A CFAR adaptive matched filter detector," *IEEE Trans. on Aerospace and Electronic Systems*, Vol. 28, January 1992, pp. 208-216
- [7] W.Chen and I.S. Reed, "A new CFAR detection test for radar," *Digital Signal Processing*, Vol. 1, no. 1., January 1991, pp. 198-214
- [8] L.Scharf and B.Friedlander, "Matched Subspace Detectors," *IEEE Trans. on Signal Processing*, vol. 42, 1994, pp.2146-2157
- [9] L.Scharf and T.-McWhorter, "Adaptive Matched Subspace Detector and Adaptive Coherence Estimators," *Proceedings of the 30th Asilomar Conference on Signals, Systems, and Computers*, Pacific Grove, CA, 1996.
- [10] L.Scharf, T.McWhorter, and L.Griffiths, "Adaptive Coherence Estimation for Radar Signal Processing," *Proceedings of the 30th Asilomar Conference on Signals, Systems, and Computers*, Pacific Grove, CA, 1996.
- [11] S. Kraut, T. McWhorter, and L.Scharf, "A canonical representation for the distributions of adaptive matched subspace detectors," *Proceedings of the 31st Asilomar Conference on Signals, Systems, and Computers*, Pacific Grove, CA, 1997.
- [12] S.Kraut and L.Scharf, "The CFAR Adaptive Subspace Detector is a Scale-Invariant GLRT," *IEEE Trans. on Signal Processing*, vol. 47 September 1999 pp.2538-2541.
- [13] S.Kraut, L.L. Scharf, and L.McWhorter, "Adaptive subspace detectors," *IEEE Trans. on Signal Processing*, vol.49 January 2001, pp. 1-16
- [14] R.Nitzberg, "An effect of range-heterogeneous clutter on adaptive Doppler filters," *IEEE Trans. on Aerospace and Electronic Systems*, vol.26, no.3, 1990 pp.475-480
- [15] K.R. Gerlach, "Outlier resistant adaptive matched filtering," *IEEE Trans. on Aerospace and Electronic Systems*, vol.38, no.3, 2002 pp.885-901
- [16] W.L. Melvin, "Space-time adaptive radar performance in heterogeneous clutter," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 36, no.2, April 2000 pp.621-633
- [17] M. Rangaswamy, J.H. Michels, B. Himed, "Statistical analysis of the non-homogeneity detector for STAP applications," *Proceedings of the National Radar Conference*, Atlanta, GA, May 2001
- [18] M. Rangaswamy and J.H. Michels, "A parametric detection algorithm for space-time adaptive processing in non-Gaussian clutter," in *Defence Applications of Signal Processing*, Eds: D. Cochran, L. White, and B. Moran, Elsevier Science B.V., Amsterdam, Netherlands, 2001
- [19] J.H. Michels, B.Himed, and M.Rangaswamy, "Performance of STAP tests in Gaussian and Compound-Gaussian Clutter," *Digital Signal Processing* vol. 10, no.4, October 2000, pp.309-324.
- [20] J.H. Michels, M.Rangaswamy, and B.Himed, "Performance of parametric and covariance based STAP tests in compound-Gaussian clutter," *Digital Signal Processing* vol. 12, no.2/3, April/July 2002, pp.307-328
- [21] F. Gini and J.H. Michels, "Performance analysis of two covariance matrix estimators in compound-Gaussian clutter," *IEEE Proceedings Part F: Radar, Sonar and Navigation*, vol.146, no.3, June 1999 pp. 133-140
- [22] F. Gini and A. Farina, "Vector subspace detection in compound-Gaussian clutter. Part I: survey and new results," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 38 no. 4 October 2002, pp. 1295 - 1311
- [23] F. Gini and A. Farina, "Vector subspace detection in compound-Gaussian clutter. Part II: performance analysis," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 38 no. 4 October 2002, pp. 1312 - 1323
- [24] M. Rangaswamy, J.H. Michels, and D.D. Weiner, "Multichannel detection for correlated non-Gaussian random processes based on innovations," *IEEE Transactions on Signal Processing*, vol. 43, no.8, August 1995, pp. 1915-1922
- [25] J.R. Roman, M. Rangaswamy, D. W. Davis, Q. Zhang, B. Himed, and J.H. Michels, "Parametric adaptive matched filter for airborne radar applications," *IEEE Transactions on Aerospace and Electronic Systems*, vol 36, no. 2, April 2000, pp. 677-692

- [26] A.L. Swindlehurst and P.Stoica, "Maximum Likelihood Methods in Radar Array Signal Processing," *Proceedings of the IEEE*, vol. 86, No. 2, February 1998, pp. 421—441
- [27] L.Timmoneri, I.K. Proudler, A.Farina, and J.G. McWhirter, "QRD-based MVDR algorithm for adaptive multipulse antenna array signal processing," *IEE Proc.F, Radar, Sonar and Navigation*, Vol. 141, No. 2 1994 pp. 93—102
- [28] J.S. Goldstein and I.S. Reed, "Theory of partially adaptive radar," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 33, no.4, October 1997, pp. 1309-1325
- [29] D.A. Pados and G.N. Karystinos, "An iterative algorithm for the computation of the MVDR filter," *IEEE Transactions on Signal Processing*, vol. 49, no.2, February 2001, pp. 290-300
- [30] I.S. Reed and Yo-Ling Gau, "An improved reduced-rank CFAR space-time adaptive radar detection algorithm," *IEEE Transactions on Signal Processing*, vol. 46, no. 8, August 1998, pp. 2139-2146
- [31] J.S. Goldstein, I.S. Reed, and P. Zulch, "Multistage partially adaptive STAP CFAR detection algorithm," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 35, no.2, April 1999, pp. 645-661
- [32] A.G. Jaffer, M.H. Baker, W.P. Balance, and J.R. Staub, "Adaptive space-time processing techniques for airborne radar," *Rome Laboratory Technical Report, RL-TR-91-162*, July 1991.
- [33] I.P. Kirsteins, and D.W. Tufts, "Adaptive detection using low rank approximation to a data matrix," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 30, no. 1, January 1994 pp. 55-67
- [34] A.M. Haimovich, "The eigencanceller: adaptive radar by eigenanalysis methods," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 32, no. 2, April 1996 pp. 532-542
- [35] C.D. Peckham, A.M. Haimovich, T.F. Ayoub, J.S. Goldstein, and I.S. Reed, "Reduced-rank STAP performance analysis," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 36, no.2, April 2000, pp. 664-676
- [36] M. Rangaswamy, F.C. Lin, and K.R. Gerlach, "Robust adaptive signal processing methods for heterogeneous radar clutter scenarios," *Proceedings of the 2003 IEEE Radar conference, Huntsville, AL, May 2003*
- [37] M. Rangaswamy, J.H. Michels, and B. Himed, "Statistical analysis of the non-homogeneity detector for STAP applications," *Digital Signal Processing*, Vol. 13, no. 4, October 2003.

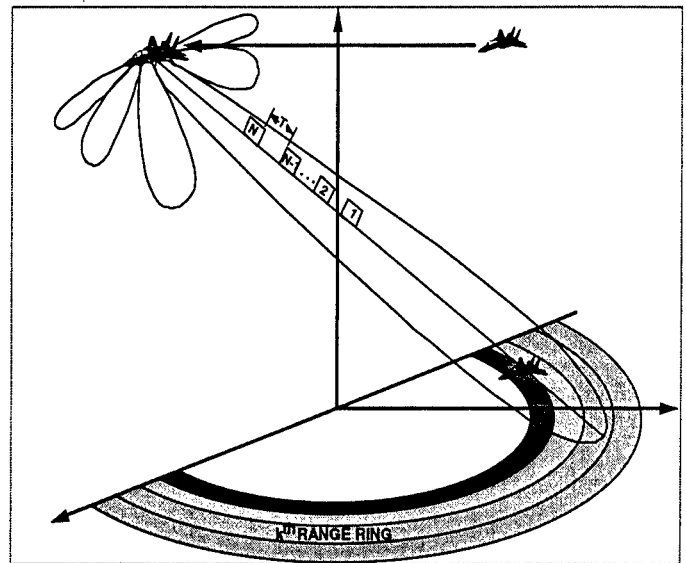


Figure 2: Airborne Radar Scenario

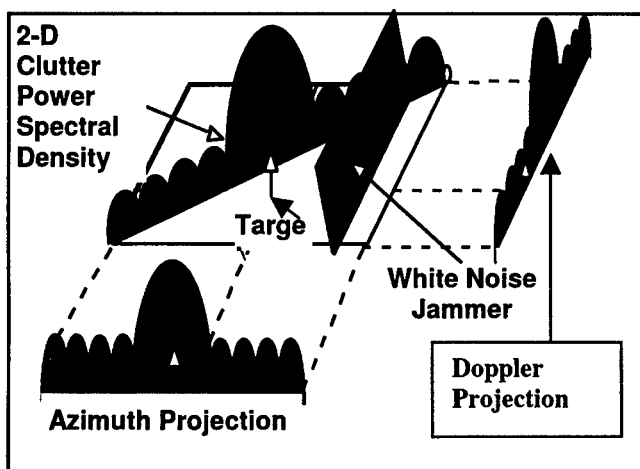


Figure 1: Power Spectrum from a Range Cell

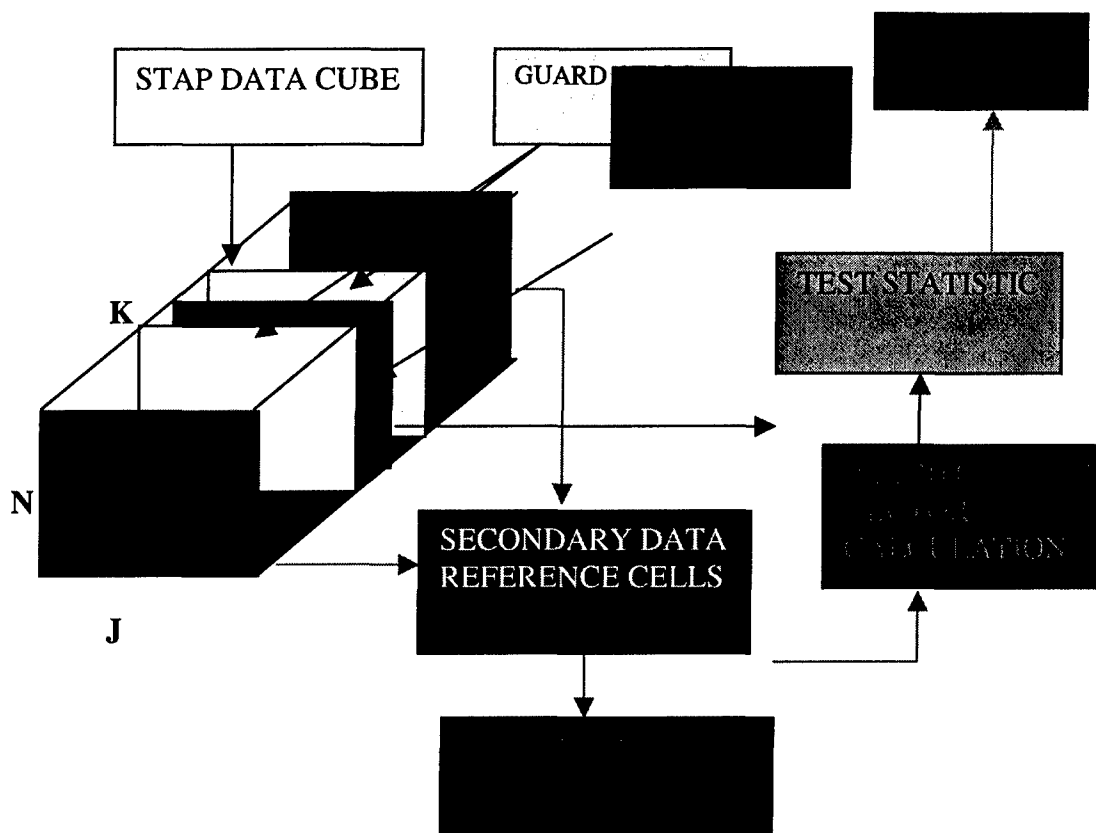


Figure 3: Classical STAP Processing

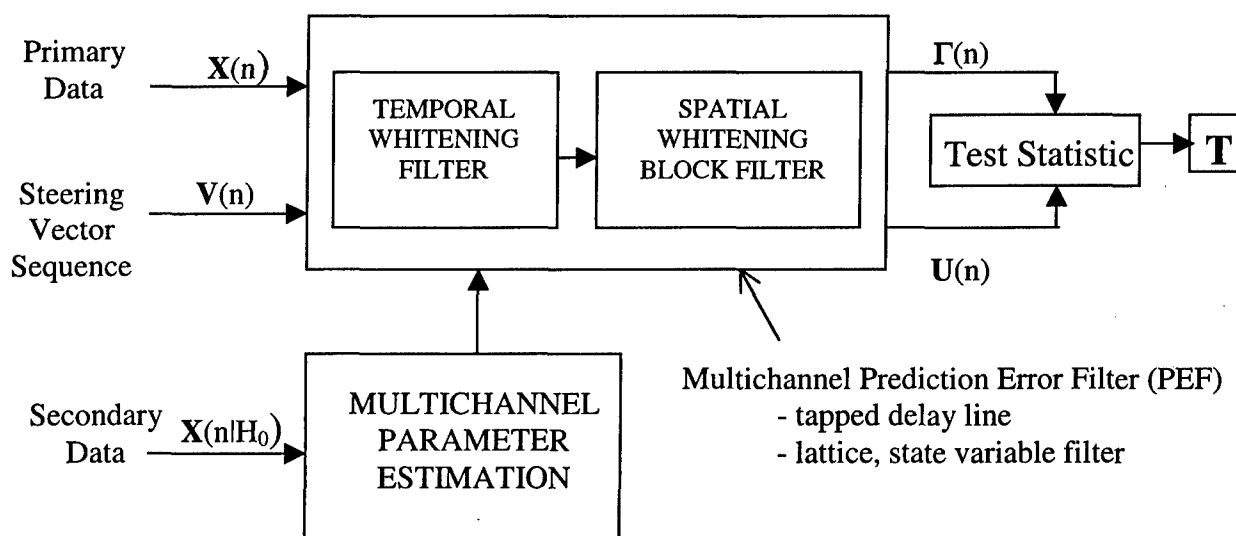


Figure 4: Parametric STAP

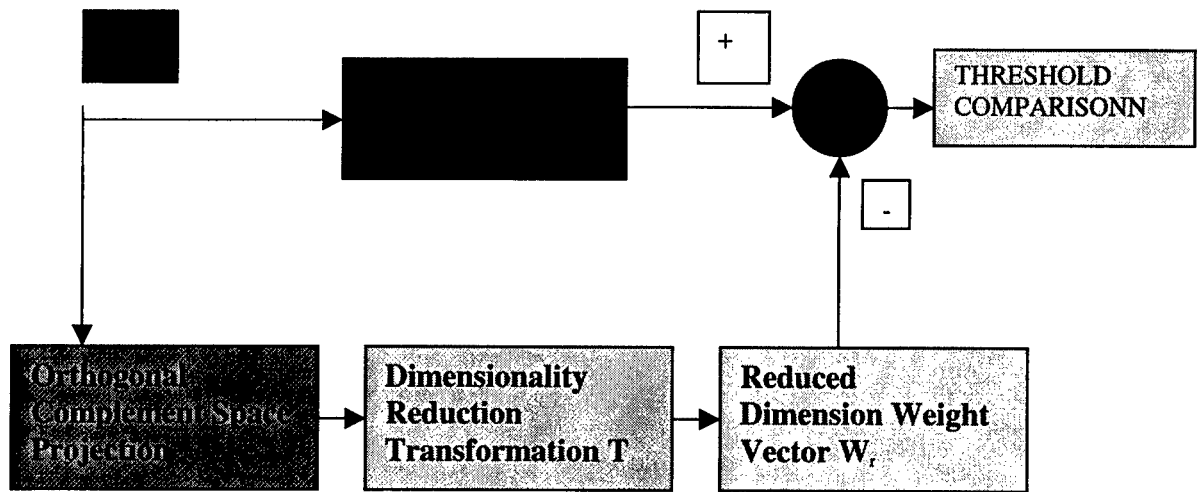


Figure 5: Reduced Rank STAP